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ABSTRACT

All parametric analysis focuses on the "synthetic" variables created by applying weights to "observed" variables, but these synthetic variables are called by different names across methods. This paper explains four ways of computing the synthetic scores in factor analysis: (1) regression scores; (2) M. S. Bartlett's algorithm (1937); (3) the Anderson-Rubin method (T. W. Anderson and H. Rubin, 1956); and (4) standardized, noncentered factor scores. A description and illustration of the derivation and utility of factor scores in multivariate analysis were undertaken. In addition, an attempt was made to explain the concept that factor scores are synthetic variables, or weighted combinations of the observed scores, and that they are similar to those in regression. An appendix contains a Statistical Package for the Social Sciences program. (Contains 6 tables and 11 references.) (Author/SLD)

Alternate Ways of Computing Factor Scores

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Paper presented at the 19th annual meeting of the Southwest Educational Research Association, New Orleans, LA, January 27, 1996.

Abstract

All parametric analysis focus on the "synthetic" variables created by applying weights to "observed" variables. These synthetic variable scores are called by different names across methods. The present paper explains four ways of computing the synthetic scores in factor analysis: (a) regression scores, (b) Bartlett's algorithm, (c) Anderson-Rubin, and (d) standardized, noncentered factor scores. A description and illustration of the derivation and utility of factor scores in multivariate analysis was undertaken. Additionally, an attempt to explain the concept that factor scores are synthetic variables, or weighted combinations of the observed scores, and are similar to yhat in regression was undertaken as well.

It is important for researchers to understand that all parametric analyses are correlational and invoke weights applied to "observed" variables to create the "synthetic" variables that are actually the focus of the analysis and yield variance-accounted-for effect sizes (Fan, 1992; Knapp, 1978; Thompson, 1991). The conventional language we use obscure these realizations and confuses those attempting to gain an understanding of parametric analyses. For example, we call the same **systems of weights** "equations" in regression, "factors" in factor analysis, "functions" or "rules" in discriminant analysis, and "functions" in canonical correlation analysis. We call the **weights** themselves "beta" weights in regression, "pattern coefficients" in factor analysis, and "standardized function coefficients" in discriminant function or canonical correlation analysis. The **synthetic scores** are called "yhat" in regression, "factor scores" in factor analysis, "discrimant scores" in discriminant analysis, and "canonical function (or variate) scores" in canonical correlation analysis.

The present paper explains in detail the synthetic scores computed in factor analysis--factor scores. Specifically, four ways of computing factor scores will be explained. These include: (a) regression scores, (b) Bartlett's algorithm, and (c) the Anderson-Rubin method (Gorsuch, 1983). A fourth alternative--standardized, noncentered factor scores (Thompson, 1993)--will also be explained.

A small data set from Holzinger and Swineford (1939) will be used to heuristically illustrate all calculations. The variables are from a battery of 301 scores on 9 achievement tests. The nine tests analyzed were:

1. Paragraph comprehension test

2. Sentence completion test

3. Word meaning test

4. Speeded addition test

5. Speeded counting of dots in shape

6. Speeded discrimination of straight and curved caps

7. Memory of target words

8. Memory of target numbers

9. Memory of object-number association targets

The purpose of factor analysis is to simplify the matrix of association by finding the underlying constructs between the variables and separating these into a smaller set of factors. The factors are latent variables that cannot be directly observed or counted or measured. Interest centers mainly on the common factors, which are interpreted with reference to the observed variables. In the present study, factor analysis indicated the presence of three factors. An examination of the observed variables indicated that these fators represent three dimensions or constructs being measured by the achievement tests--comprehension, speed, and memory.

An initial step in factor analysis is the computation of a matrix of association coefficients from the raw data matrix. In the present study, a correlation matrix was used, however, any matrix of association may be used. Examination of the correlation matrix derived from the heuristic data set, presented in Table 1, indicates a symmetric matrix with 9 rows and 9 columns, and with redundant off-diagonal triangles (R_{vxv}). For any symmetric matrix, such as a variable intercorrelation matrix, there is an inverse of this R matrix called R_{vxv}^{-1}, such that $R_{vxv} R_{vxv}^{-1} = I_{vxv}$ (Gorsuch, 1983). I_{vxv} is called an Identity matrix, because any matrix times I equals the same matrix you started with. The I matrix in matrix algebra is equivalent in regular algebra to multiplying by 1. When the symmetric matrix is R_{vxv}, saying R is an I matrix is the same as saying the variables are perfectly uncorrelated. When factors are first extracted, the factor correlation matrix is always an Identity matrix. After an orthogonal (orthogonal = uncorrelated) rotation, the factor correlation matrix is still an Identity matrix. However, in an oblique (oblique = correlated) rotation, the factor correlation matrix is no longer an Identity matrix.

Insert Table 1 About Here

The interpretation of a factor is based upon the variables that are and are not related to that factor. When the factors are **uncorrelated**, one matrix summarizes the factor-variable relationships and is called the **factor pattern/structure coefficient matrix**. However, just as in regression, in some cases the beta weights and structure coefficients are the same, but when the two are not equal, both must be interpreted (Thompson, 1992). When the factors are not perfectly uncorrelated, the contribution of each variable to the interpretation will differ depending upon which of several factor matrices are examined, thus both the factor structure coefficient matrix and the factor pattern coefficient matrix must be interpreted. The factor structure matrix presents the bivariate correlations between the observed variable scores of the n people with the latent variable or factor scores of the same n people. This is the most commonly used matrix for the interpretation of the factors.

The factor pattern matrix is the weight matrix used to calculate factor scores. Factor scores are synthetic variables, or weighted combinations of the observed scores of the n people, and are similar to yhat scores in regression. In regression, you use the beta weights, not the structure coefficients, to compute yhat scores. In factor analysis, if the pattern and structure coefficients are not equal, you must use the pattern coefficients to compute factor scores. Factor scores are the predicted scores of the n individuals on the f factors. The factor pattern matrix represents only the **unique contribution** of each factor to each variable, arbitrarily removing redundancies among any correlated variables originating from the same factor. Factor scores are used to relate the factors to other variables. They reflect the impact of the factor on each variable.

The best procedure for computing factor scores would have the following characteristics (McDonald & Burr, 1967):

> First, the common factor scores should have a high correlation with the factors they are attempting to measure. Second, common factor scores should be conditionally unbiased estimates of the true factor scores. Third, in the case of orthogonal factors, the factor scores estimated from one factor should have zero correlations with all other factors. Fourth, if the factors are orthogonal, the common factor scores should correlate zero with each other. (Gorsuch, 1983, p. 260)

The rank of a factor pattern/structure matrix is the number of variables by the number of factors, $_{v \times f}$. The rank of a correlation matrix is the number of variables by the number of variables, $_{v \times v}$. The rank of a factor correlation matrix is the number of factors by the number of factors, $_{f \times f}$. According to Gorsuch (1983), in order to multipy matrices, the number of columns in the left matrix must be equal to the number of rows in the adjacent matrix to the right (called "conformability"). The resulting matrix in a multiplication has rows equal to the number of rows in the leftmost matrix and columns equal to the number of columns in the rightmost matrix. In the case of the heuristic data set used in this study, the factor score matrix will indicate the synthetic scores of 301 (n) individuals on the 3 (f) factors, as noted in Table 2.

Insert Table 2 About Here

Four ways to compute factor scores are explored in the present study: (a) the Regression Algorithm, (b) Bartlett, (c) Anderson-Rubin and (d) the BTF score. The analysis was performed with the SPSS commands presented in Appendix A. A factor score is a new variable, a weighted combination of the scores on each of the variables (McMurray, 1987). The Regression Algorithm derives the factor scores based on Z-scores and uses the matrix formula ($Z\ R^{-1}\ P = F$), where Z is the Zscore matrix, R^{-1} is the inverse of the correlation matrix, P is the pattern coefficient matrix, and F is the factor score matrix. Due to widespread familiarity with multiple regression approaches, and the availability of programs, regression estimates are popular factor scores (Gorsuch, 1983, p. 262).

Bartlett (1937) suggested a least-squares procedure to minimize the sum of squares of the unique factors over the range of the variables. Bartlett's procedure is intended to keep the noncommon factors "in their place" so that they are used only to explain the discrepancies between the observed scores and those reproduced from the common factors (p. 264). Bartlett's procedure leads to high correlations between the factor scores and the factors that are being estimated.

Anderson and Rubin (1956) proceeded in the same manner as Bartlett except they added the condition that the factor scores were required to be orthogonal, resulting in a more complex equation than Bartlett's. The Anderson-Rubin equation produces factor estimates whose correlations form an identity matrix (p. 265).

However, these three (Regression, Bartlett, Anderson-Rubin) algorithms yield factor scores that are in Zscore form (each set of factor scores has a mean of zero and a standard deviation of one). The result does not allow comparison of the mean factor score on any given factor with the mean on other factors for the same data set. The BTF score is based on Thompson (1983), and yields a standardized, noncentered factor score, which allows comparisons of Fscore means, as illustrated in Table 3.

Insert Table 3 About Here

The variables are converted to Zscore form, then the original variable means are added back onto the Zscores, so that central tendency information is retrieved, then multiplied by the inverse of the correlation matrix and by the pattern matrix as in the original Regression algorithm ($((Z+\text{mean})\ R^{-1}\ P = F)$. Thus, in this procedure, the raw data are transformed only by the division of each raw score by the standard deviation of the variable, with the result that each variable has a standard deviation of one, but a non-zero mean. This allows the comparison of the mean factor scores across factors to make judgments regarding the relative importance of given factors. All four types of factor scores for Factor 1 are shown in Table 4, allowing a comparison of the factor score relationships for Factor 1.

Insert Table 4 About Here

Although the means of the four types of factor scores are not equal, when the correlations among them are examined, all of the factor scores correlate perfectly with themselves, and are perfectly uncorrelated with all other factor scores (see Table 5). This indicates that the factor scores conform to the McDonald and Burr (1967) specifications.

Insert Table 5 About Here

As previously indicated, the factor structure matrix is the bivariate correlation between the n observed variables with the n latent variables. Thus, a structure coefficient indicates the correlation between the scores on the observed variables (n=301) and the factor scores (n=301). For heuristic purposes, the following comparison between the correlation of the n observed variables and the n factor scores and the factor structure matrix demonstrates this, as can be noted from Table 6.

Insert Table 6 About Here

Summary

Factor scores, in addition to being used to determine the relationship between the observed and latent variables, can also be used in Anovas, Manovas, or other statistical analyses of differences between means in exploratory studies to provide additional information about what is going on with the data. A major problem with multivariate techniques such as MANOVA and canonical correlations when using a large number of variables is the difficulty of interpretation. Separate factor analysis of independent and dependent variables followed by an analysis of the factor scores results in greater interpretability because each variable set is orthogonalized and fewer variables are in the analysis (Gorsuch, 1983, p. 365). However, factor scores would generally not be used developing theories about structure.

Through factor analysis, a complex analysis of multiple variables can be simplified, and patterns among the variables can be made evident. The purpose of the present paper was an attempt to describe and illustrate the derivation and utility of factor scores in multivariate analyses. An attempt to simplify and explain the concept that factor scores are synthetic variables, or weighted combinations of the observed scores, and are similar to yhat in regression was undertaken as well. It is hoped that this analysis will allow the interested reader to come to the realization that all parametric analyses are (a) correlational and (b) invoke weights applied to observed variables to create synthetic variables; a concept that is often obscured by the confusing practice of using different language and names to describe the same concepts.

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Table 1
Intercorrelation Matrix for Holzinger & Swineford (1939) Data.

	T6	T7	T9	T10	T12	T13	T14
T6	1.00000						
T7	.73317	1.00000					
T9	.70448	.71996	1.00000				
T10	.17383	.10204	.12110	1.00000			
T12	.10690	.13867	.14961	.48676	1.00000		
T13	.20785	.22747	.21416	.34065	.44902	1.00000	
T14	.22242	.15355	.17205	.09293	.03850	.13608	1.00000
T15	.06945	-.01889	.05191	.10896	.07786	.07205	.39674
T17	.14527	.09230	.14554	.33090	.22992	.19811	.35468

Table 2
Correlations Between Factor Scores and Variables.

	FSCORE1	FSCORE2	FSCORE3
T6	.8863 (301) P= .000	.0944 (301) P= .102	.1191 (301) P= .039
T7	.9058 (301) P= .000	.0987 (301) P= .087	-.0024 (301) P= .967
T9	.8817 (301) P= .000	.1062 (301) P= .066	.0739 (301) P= .201
T10	.0350 (301) P= .546	.7727 (301) P= .000	.1449 (301) P= .012
T12	.0457 (301) P= .430	.8356 (301) P= .000	.0089 (301) P= .878
T13	.1974 (301) P= .001	.6964 (301) P= .000	.0558 (301) P= .335
T14	.1858 (301) P= .001	-.0215 (301) P= .710	.7871 (301) P= .000
T15	-.0444 (301) P= .443	.0153 (301) P= .792	.7881 (301) P= .000
T17	.0481 (301) P= .405	.3601 (301) P= .000	.6422 (301) P= .000

Table 3
A Comparison of Factor Scores using standardized, noncentered factor scores (Thompson, 1993).

FSBT1	FSBT2	FSBT3
5.06	133.94	109.06
3.99	134.99	110.03
3.31	132.[illegible]9	109.12
5.30	132[illegible]4	109.24
4.86	134[illegible]5	111.31
3.58	135.34	108.67
5.85	134.62	105.89
4.94	132.56	110.89
5.52	134.28	110.87
5.34	132.77	109.83
4.31	133.40	109.64
5.09	132.81	110.25
4.76	134.48	109.58
5.33	133.22	110.12
6.46	133.22	108.94
4.71	132.78	109.20
4.27	134.38	109.26
4.06	133.81	108.34
6.51	133.40	108.23
5.25	132.90	110.71
3.63	132.84	110.14
4.35	134.50	109.85
6.03	132.98	109.00
4.06	134.74	110.00
4.28	133.33	109.32

Number of cases read: 25   Number of cases listed: 25

Number of valid observations (listwise) = 301.00

Variable	Mean	Std Dev	Minimum	Maximum	N	Label
FSBT1	5.08	1.00	2.85	8.00	301	COMPREHENSION hard
FSBT2	133.72	1.00	131.01	136.84	301	SPEED hard
FSBT3	109.84	1.00	105.89	112.43	301	MEMORY hard

Table 4
A Comparison of Factor Scores on Factor 1.

FSCOR1	FSCR1	FSHARD1	FSBT1
-.01769	-.01769	-.02	5.06
-1.08497	-1.08497	-1.08	3.99
-1.76143	-1.76143	-1.76	3.31
.22325	.22325	.22	5.30
-.21953	-.21953	-.22	4.86
-1.49075	-1.49075	-1.49	3.58
.77463	.77463	.77	5.85
-.14014	-.14014	-.14	4.94
.44679	.44679	.45	5.52
.26183	.26183	.26	5.34
-.76789	-.76789	-.77	4.31
.01686	.01686	.02	5.09
-.31066	-.31066	-.31	4.76
.25731	.25731	.26	5.33
1.38274	1.38274	1.38	6.46
-.36791	-.36791	-.37	4.71
-.80582	-.80582	-.81	4.27
-1.01354	-1.01354	-1.01	4.06
1.43051	1.43051	1.43	6.51
.17900	.17900	.18	5.25
-1.44236	-1.44236	-1.44	3.63
-.72166	-.72166	-.72	4.35
.95367	.95367	.95	6.03
-1.01925	-1.01925	-1.02	4.06
-.79077	-.79077	-.79	4.28

Table 5
Correlations Between Regression, Bartlett, Anderson-Rubin, and BT Factor Scores.

	FSCOR1	FSCOR2	FSCOR3	FSCR1	FSCR2	FSCR3	FSHARD1	FSHARD2	FSHARD3	FSBT1	FSBT2	FSBT3
[illegible]	[illegible]	[illegible]	.0000	1.0000	[illegible]	.0000	1.0000	.0000	.0000	[illegible]	.0000	.0000
[illegible]	[illegible]	[illegible]	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000
[illegible]	[illegible]	[illegible]	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000
[illegible]	1.0000	[illegible]	[illegible]	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000
[illegible]	[illegible]	[illegible]	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000
[illegible]	[illegible]	[illegible]	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000
[illegible]	[illegible]	.0000	.0000	[illegible]	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000
[illegible]	[illegible]	[illegible]	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000
[illegible]	[illegible]	[illegible]	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000	.0000	.0000	1.0000
[illegible]	[illegible]	[illegible]	.0000	1.0000	[illegible]	.0000	1.0000	[illegible]	.0000	1.0000	.0000	.0000
[illegible]	[illegible]	[illegible]	.0000	.0000	1.0000	.0000	.0000	[illegible]	.0000	.0000	1.0000	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0000	.0000	1.0000	.0000	.0000	[illegible]

Table 6
A Comparison of the Rotated Factor Structure Matrix and a Correlation Between Variables and Factor Scores.

Rotated Factor Matrix:

	Factor 1	Factor 2	Factor 3
T6	.88632	.09442	.11913
T7	.90577	.09867	-.00241
T9	.88167	.10625	.07386
T10	.03496	.77269	.14494
T12	.04570	.83563	.00892
T13	.19744	.69641	.05581
T14	.18580	-.02151	.78712
T15	-.04440	.01528	.78815
T17	.04814	.36013	.64217

Correlation Matrix:

	FSCORE1	FSCORE2	FSCORE3
T6	.8863	.0944	.1191
T7	.9058	.0987	-.0024
T9	.8817	.1062	.0739
T10	.0350	.7727	.1449
T12	.0457	.8356	.0089
T13	.1974	.6964	.0558
T14	.1858	-.0215	.7871
T15	-.0444	.0153	.7881
T17	.0481	.3601	.6422

APPENDIX A

SPSS Program

```
DATA LIST
  FILE 'a:HOLZINGR.DTA' FIXED RECORDS=2 TABLE
  /1 id 1-3 sex 4-4 ageyr 6-7
  agemo 8-9 t1 11-12 t2 14-15 t3 17-18 t4 20-21 t5 23-24 t6 26-27 t7 29-30 t8
  32-33 t9 35-36 t10 38-40 t11 42-44 t12 46-48 t13 50-52 t14 54-56 t15 58-60
  t16 62-64 t17 66-67 t18 69-70 t19 72-73 t20 74-76 t21 78-79 /2 t22 11-12
  t23 14-15 t24 17-18 t25 20-21 t26 23-24 .
EXECUTE.
COMPUTE SCHOOL=1.
IF (ID GT 200)SCHOOL=2.
IF (ID GE 1 AND ID LE 85)GRADE=7.
IF (ID GE 86 AND ID LE 168)GRADE=8.
IF (ID GE 201 AND ID LE 281)GRADE=7.
IF (ID GE 282 AND ID LE 351)GRADE=8.
IF (ID GE 1 AND ID LE 44)TRACK=2.
IF (ID GE 45 AND ID LE 85)TRACK=1.
IF (ID GE 86 AND ID LE 129)TRACK=2.
IF (ID GE 130)TRACK=1.
PRINT FORMATS SCHOOL TO TRACK(F1.0).
VALUE LABELS SCHOOL(1)PASTEUR (2) GRANT-WHITE/
  TRACK (1)JUNE PROMOTIONS (2)FEB PROMOTIONS/.
VARIABLE LABELS T1 VISUAL PERCEPTION TEST FROM SPEARMAN VPT, PART III
  T2 CUBES, SIMPLIFICATION OF BRIGHAM'S SPATIAL RELATIONS TEST
  T3 PAPER FORM BOARD--SHAPES THAT CAN BE COMBINED TO FORM A TARGET
  T4 LOZENGES FROM THORNDIKE--SHAPES FLIPPED OVER THEN IDENTIFY TARGET
  T5 GENERAL INFORMATION VERBAL TEST
  T6 PARAGRAPH COMPREHENSION TEST
  T7 SENTENCE COMPLETION TEST
  T8 WORD CLASSIFICATION--WHICH WORD NOT BELONG IN SET
  T9 WORD MEANING TEST
  T10 SPEEDED ADDITION TEST
  T11 SPEEDED CODE TEST--TRANSFORM SHAPES INTO ALPHA WITH CODE
  T12 SPEEDED COUNTING OF DOTS IN SHAPE
  T13 SPEEDED DISCRIM STRAIGHT AND CURVED CAPS
  T14 MEMORY OF TARGET WORDS
  T15 MEMORY OF TARGET NUMBERS
  T16 MEMORY OF TARGET SHAPES
  T17 MEMORY OF OBJECT-NUMBER ASSOCIATION TARGETS
  T18 MEMORY OF NUMBER-OBJECT ASSOCIATION TARGETS
  T19 MEMORY OF FIGURE-WORD ASSOCIATION TARGETS
  T20 DEDUCTIVE MATH ABILITY
  T21 MATH NUMBER PUZZLES
  T22 MATH WORD PROBLEM REASONING
  T23 COMPLETION OF A MATH NUMBER SERIES
  T24 WOODY-MCCALL MIXED MATH FUNDAMENTALS TEST
  T25 REVISION OF T3--PAPER FORM BOARD
  T26 FLAGS--POSSIBLE SUBSTITUTE FOR T4 LOZENGES.
SUBTITLE 'FACTOR 9 VARIABLES INTO 3 FACTORS ##############'.
FACTOR VARIABLES=T6 T7 T9 T10 T12 T13 T14 T15 T17/
  PRINT=ALL/PLOT=EIGEN/
  CRITERIA=FACTORS(3)/EXTRACTION=PC/ROTATION=VARIMAX/
  SAVE=REG(ALL FSCORE).
VARIABLE LABELS FSCORE1 'COMPREHENSION reg'
  FSCORE2 'SPEED reg'
  FSCORE3 'MEMORY reg'.
SUBTITLE '1.  Show what structure coefficients are $$$$$$$$$$'.
CORRELATIONS VARIABLES T6 T7 T9 T10 T12
  T13 T14 T15 T17 WITH FSCORE1 FSCORE2 FSCORE3.
SUBTITLE '3a. Factor scores  BARTLETT method ##############'.
```

```
FACTOR VARIABLES=T6 T7 T9 T10 T12 T13 T14 T15 T17/
  CRITERIA=FACTORS(3)/EXTRACTION=PC/ROTATION=VARIMAX/
  SAVE=BART(ALL FSCOR).
VARIABLE LABELS FSCOR1 'COMPREHENSION bart'
  FSCOR2 'SPEED bart'
  FSCOR3 'MEMORY bart'.
SUBTITLE '3b. Factor scores  ANDERSON-RUBIN ##############'.
FACTOR VARIABLES=T6 T7 T9 T10 T12 T13 T14 T15 T17/
  CRITERIA=FACTORS(3)/EXTRACTION=PC/ROTATION=VARIMAX/
  SAVE=AR(ALL FSCR).
VARIABLE LABELS FSCR1 'COMPREHENSION ar'
  FSCR2 'SPEED ar'
  FSCR3 'MEMORY ar'.
SUBTITLE '4a.  Compute z-score ******************'.
DESCRIPTIVES VARIABLES=T6 TO T17/save.
SUBTITLE '4b.  Prove z-scores are z-scores *******'.
DESCRIPTIVES VARIABLES=ZT6 to ZT17.
SUBTITLE '4c.  Compute regression factor scores hard way *****'.
COMPUTE FSHARD1=(.36704*ZT6)+(.38244*ZT7)+(.36666*ZT9)+(-.06692*ZT10)+
  (-.05997*ZT12)+(.01598*ZT13)+(.03449*ZT14)+(-.06859*ZT15)+
  (-.05238*ZT17).
COMPUTE FSHARD2=(-.04461*ZT6)+(-.03002*ZT7)+(-.03228*ZT9)+(.41486*ZT10)+
  (.46499*ZT12)+(.36683*ZT13)+(-.12069*ZT14)+(-.07829*ZT15)+
  (.12392*ZT17).
COMPUTE FSHARD3=(.01004*ZT6)+(-.06802*ZT7)+(-.019496*ZT9)+(-.00034*ZT10)+
  (-.09376*ZT12)+(-.05743*ZT13)+(.48589*ZT14)+(.49629*ZT15)+
  (.359058*ZT17).
VARIABLE LABELS FSHARD1 'COMPREHENSION hard'
  FSHARD2 'SPEED hard'
  FSHARD3 'MEMORY hard'.
SUBTITLE '5.  Compute Thompson factor scores ^^^^^^^^^^^'.
COMPUTE TT6=ZT6+9.18272.
COMPUTE TT7=ZT7+17.36213.
COMPUTE TT9=ZT9+15.29900.
COMPUTE TT10=ZT10+96.27575.
COMPUTE TT12=ZT12+110.54153.
COMPUTE TT13=ZT13+193.46844.
COMPUTE TT14=ZT14+175.15282.
COMPUTE TT15=ZT15+90.00997.
COMPUTE TT17=ZT17+8.23256.
COMPUTE FSbt1=(.36704*TT6)+(.38244*TT7)+(.36666*TT9)+(-.06692*TT10)+
  (-.05997*TT12)+(.01598*TT13)+(.03449*TT14)+(-.06859*TT15)+
  (-.05238*TT17).
COMPUTE FSbt2=(-.04461*TT6)+(-.03002*TT7)+(-.03228*TT9)+(.41486*TT10)+
  (.46499*TT12)+(.36683*TT13)+(-.12069*TT14)+(-.07829*TT15)+
  (.12392*TT17).
COMPUTE FSbt3=(.01004*TT6)+(-.06802*TT7)+(-.019496*TT9)+(-.00034*TT10)+
  (-.09376*TT12)+(-.05743*TT13)+(.48589*TT14)+(.49629*TT15)+
  (.359058*TT17).
VARIABLE LABELS FSbt1 'COMPREHENSION hard'
  FSbt2 'SPEED hard'
  FSbt3 'MEMORY hard'.
SUBTITLE '6.  Show factor score relationships &&&&&&&&&&&&&'.
LIST VARIABLES=FSCORE1 FSCOR1 FSCR1 FSHARD1 FSBT1/CASES=25.
DESCRIPTIVES VARIABLES=FSCORE1 to FSCR3 FSHARD1 to FSHARD3 FSbt1 to FSbt3.
CORRELATIONS VARIABLES=FSCORE1 to FSCR3 FSHARD1 to FSHARD3 FSbt1 to FSbt3.
LIST VARIABLES=FSBT1 FSBT2 FSBT3/CASES=25.
DESCRIPTIVES VARIABLES=FSbt1 to FSbt3.
```